Application of Excel simulation for tuning of level control for three tank system

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Abstract
In process control courses, the issues of controller design and stability analysis are often emphasised. However, in practice, the following three issues are usually more important: (1) how well the plant can be controlled, (2) what strategy should be used and (3) how the plant should be changed to improve control. These issues have been addressed by Skogestad (1996), but less sophisticated methods are sometimes needed for a better analysis. This document outlines a simple procedure for analysing the input-output controllability of single-input single-output (SISO) systems. This procedure is applied to a three tank process which must be redesigned to obtain acceptable controllability. It is found that more or less heuristic design rules given in the literature follow directly. The key steps in the analysis are to consider disturbances and to scale the variables properly. It is suggested that most of the material presented in this paper is suitable for inventory control. The paper was inspired by the system analysis course presented at the University of Pretoria’s Industrial Engineering Department.

Keywords: Proportional Controller, System dynamics, System delay

1. Introduction
Most liquid levels represent material inventory used as surge capacity. In these cases, it is relatively unimportant where the level is, as long as it is between a maximum and minimum level.

Therefore, proportional controllers are often used on level loops to give smooth changes in flow rates and to filter out fluctuations in flow rates to downstream units. Although the controllers have only three parameters, it is not easy to find suitable values for them without a systematic procedure. In fact, a visit to a process plant will usually show that a large number of these controllers are poorly tuned.

In many process plants, expensive and sophisticated software is used for controller tuning, but a more accessible and easier alternative is to use Excel Software. This report discusses the model formulation and the solution method of Proportional Controller Tuning for a three tank system.

2. Problem Statement
A classic control problem in many industries is the control of inventory or “Level Control”, which can be stated as follows:
*Given a controller type (e.g. Proportional, Proportional Integral or PID) for a system, determine the tuning constants that give a desired closed loop performance.*
3. Approach

a. Mathematical Model: A tank system model

A simple difference or state equation is written for each of the three tanks; this equation considers the previous state and uses the rate of change over a given small time internal to predict the current or present state.

\[ \Delta L_{t+1} = L_t + \text{Rate} \times \Delta t \]

or

\[ \Delta L_{t+1} = L_t + \Delta t \times \left( \frac{dX}{dt} \right) \]

It must further be stated that the rate of change for any system depends on the rate of input and output, as shown by Equation 2:

\[ \frac{dX}{dt} = F_{in} - F_{out} \]

This equation is a simple, first order equation and can easily be solved using the simple Euler equation, which can be written in discreet form as given below:

\[ (dX_{dot})_n = F_{in} - F_{out} \]

Using above equations, then Equation 1 can be written as follows:

\[ L_{t+1} = L_t + \Delta t \times (F_{in} - F_{out})_t \]

or

\[ \Delta L_{t+1} = L_t + \Delta t \times \left( \frac{dX}{dt} \right) \]


b. Closed loop Dynamic Model

Figure 2 below shows the closed loop system for the three tanks, where the output of second tank regulates the intake and the output is controlled by the third tank. The intake is regulated accordingly to the set value in Tank two, while Tank 3’s output is also controlled. There is a small disturbance incorporated into this model as well.

c. Controller Algorithm

The Algorithm used in this model is a typical proportional feedback where the error is bumped up by a typical constant normally referred to as proportional gain.

\[ C_{A0} = C_{AM} + C_{AD} \]

\[ C_{AM} = B_{act} \pm K_c E(T) + \frac{K_c}{\tau} \int E(t) dt \]

\[ E(T) = (\text{Setpoint} - \text{Actual}) \]


d. Time Delay or “Dead time”

As dead time or transportation lag is frequently encountered in many industrial systems, the problem is handled by using equation 6, which states that a process stream flowing through a pipe and taking D minutes in it, will emerge from the other end D minutes later. This means that the output of the process must be delayed by D minutes or years. Easier said than done.

\[ f_t = f_{t-D} \]
Transportation delays are easier to handle in Laplace and frequency domain than in time domain, because the output has to be computed then delayed. The time delay adds a phase difference in the frequency domain that is not easily recognisable in the time domain.

4. Excel Based Solution

The Excel solution method is based on the solution of equation 4 applied for all three tanks at given time t. The solution method is as follows:

• The equations are first solved for an open loop system that has a steadier state value than a closed loop system.
• The closed loop solution is solved by substituting equation 5 into equation 4 as an input, as the output of the second stage determines the control strategy for that particular system.
• The challenge of output from consecutive tanks, however, is solved by recognising that the present output has to be delayed for a predetermined time period.
• For a three stage system, the equations are solved simultaneously with the output of system one determining the input of the second system and so on.

5. Results and Analysis

a. Three tank level dynamic response

Figure 3 shows the three tank dynamic level response for the given initial goal and the following controller setting of 5 and –5 respectively.

b. High System feedback constants

Due to the three tanks being high order, high feedback gains lead to an unstable system response. This can be seen with a typical feedback gain of 20 for the first two tanks, while keeping the gain of Tank 3 at –5. Figure 4 shows this typical system response for the three tanks in series.

c. System flow output

Figure 5 shows the three tanks input and output respectively, where the input flow to Tank 1 is approximately 350 m³/hr. This is vital for proper valve design to avoid the input system saturation, where the valve opens 100% but the desired flow cannot be achieved.

d. System delay output

Figure 6 shows a tank-delayed output of 2 minutes, achieved by using the following delay steps. Based on the \( \delta t = 0.02 \) and the time delays of two minutes, however, the delay steps in this case is now 200 and not 100, as flow must take some time in the tank.
e. **Tank 2 Output**

The output shown in figure 6 is for Tank 1, while Tank 2 output is shown in figure 7, where the total delay time is 4 minutes.

![Figure 7. Tank 1 delayed output](image)

f. **System Response for high controller gain setting**

The system can be made to be dynamically unstable by setting the controller gain for Tank 2 very high. This is shown in Figure 8 where the unsuspecting Control Engineer has set the gain at 50.

![Figure 8. High Gain setting of 50 for a feedback system](image)

6. **Conclusions**

The following conclusions can be drawn from the analysis of this system:
- A high feedback gain leads to process instability.
- Dynamic system analysis is vital for process analysis and design as it avoids system saturation.
- The system time delays must always be included in the system analyses even though its significance can be understood in the frequency domain.
- This system must be designed for a maximum holding peak capacity of 300 m$^3$ and 110 m$^3$ for Tank 1 and Tank 2 respectively. The maximum peak flow must also be accounted for using the feedback constant of 5 and -5 respectively.

7. **References**


